Selected Problems from Pitman's "Probability" Text

Statistics 20, Nathan Ross, Spring 2010

1.6.6

Suppose you roll a fair six-sided die repeatedly until the first time you roll a number that you have rolled before.

1. For each $r = 1, 2, \ldots$ calculate the probability p_r that you roll exactly r times.
2. Without calculation, write down the value of $p_1 + p_2 + \cdots + p_{10}$. Explain.
3. Check that your calculated values of p_r have this value for their sum.

1.r.4

There are two boxes.

Box 1 contains 2 red balls and 3 black balls.

Box 2 contains 8 red balls and 12 black balls.

One of the two boxes is picked at random, and then a ball is picked at random from the box.

1. Is the color of the ball independent of which box is chosen?
2. What if there were 10 black balls rather than 12 in Box 2, but the other numbers were the same?

1.r.11

A hat contains n coins, f of which are fair, and b of which are biased to land heads with probability 2/3. A coin is drawn from the hat and tossed twice. The first time it lands heads, and the second time it lands tails. Given this information, what is the probability that it is a fair coin?

3.4.11

Suppose that A tosses a coin which lands heads with probability p_A, and B tosses one which lands heads with probability p_B. They toss their coins simultaneously over and over again, in a competition to see who gets the first head. The one to get the first head is the winner, except that a draw results if they get their first heads together. Calculate:

1. P(A wins);
2. P(B wins);
3. P(draw);
4. the distribution of the number of times A and B must toss.