Binomial tail bounds

Show the following lemmas, interpreting $0\log(0) = 0$.

Lemma 1. *Let $X \sim \mathrm{Bi}(m, q)$ with $m \in \mathbb{N}$ and $0 < q < 1$.*

1. *If $0 < x \le m(1-q)$,*

$$\mathbb{P}(X - mq \ge x) \le \exp\left\{-m\left[\left(1 - \tfrac{x}{m} - q\right)\log\left(\frac{1 - \frac{x}{m} - q}{1-q}\right) + \left(\tfrac{x}{m} + q\right)\log\left(\frac{\frac{x}{m} + q}{q}\right)\right]\right\}.$$

2. *If $-mq \le x < 0$,*

$$\mathbb{P}(X - mq \le x) \le \exp\left\{-m\left[\left(1 - \tfrac{x}{m} - q\right)\log\left(\frac{1 - \frac{x}{m} - q}{1-q}\right) + \left(\tfrac{x}{m} + q\right)\log\left(\frac{\frac{x}{m} + q}{q}\right)\right]\right\}.$$

Lemma 2 (Roch, Exercise 2.6)**.** *Let $X \sim \mathrm{Bi}(m, q)$ with $m \in \mathbb{N}$ and $0 < q < 1$, and $\mu := \mathbb{E}[X] = mq$. For any $0 < \delta < 1$,*

$$\mathbb{P}\big(X \ge (1+\delta)\mu\big) \le \left(\frac{e^{\delta}}{(1+\delta)^{(1+\delta)}}\right)^{\mu} \le \exp\{-\mu\delta^2/3\}, \tag{1}$$

$$\mathbb{P}\big(X \le (1-\delta)\mu\big) \le \left(\frac{e^{-\delta}}{(1-\delta)^{(1-\delta)}}\right)^{\mu} \le \exp\{-\mu\delta^2/2\}. \tag{2}$$